Complex Structure of Kerr Geometry and Rotating "Photon Rocket" Solutions.

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Abstract

In the frame of the Kerr-Schild approach, we obtain a generalization of the Kerr solution to a nonstationary case corresponding to a rotating source moving with arbitrary acceleration. Similar to the Kerr solution, the solutions obtained have the geodesic and shear free principal null congruence. The current parameters of the solutions are determined by a complex retarded-time construction via a given complex worldline of source. The real part of the complex worldline defines the values of the boost and acceleration while the imaginary part controls the rotation. The acceleration of the source is accompanied by a lightlike radiation along the principal null congruence. The solutions obtained generalize to the rotating case the known Kinnersley class of the "photon rocket" solutions.

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1 Introduction

The nonstationary gravitational fields from moving sources can be described by a retarded-time scheme which is a generalization of the Lienard-Wiechard construction of classical electrodynamics. In particular, the known Kinnersley "photon rocket" solutions [1, 2] represent the nonstationary generalizations of the Schwarzschild black hole. The parameters of the Kinnersley

solution are determined by a given worldline of a moving source and are expressed via the retarded-time position, boost and acceleration of the source. Due to acceleration, the Kinnersley solutions are accompanied by a lightlike radiation.

In this work we generalize this solution to the rotating case corresponding to the twisting Kerr geometry.

Our treatment is based on the Newman complex representation of the Kerr geometry [3], which is generated by a complex world line of source $x_0(\tau)$ moving in CM^4. This representation was used to construct the retarded-time scheme [3] and to obtain solutions of the linearized Einstein equations [4].

In the present work we construct such a retarded-time scheme in the Kerr-Schild formalism [5]. It allows us to use the effective Kerr theorem [2, 6] and obtain exact nonstationary solutions having the twisting, geodesic and shear free Kerr congruence. These solutions represent a natural generalization of the Kinnersley class.

Complex structure of Kerr geometry plays a central role in our treatment. We obtain a link between the Kerr theorem and the complex worldline of the Kerr source and determine the Kerr congruence as a complex retarded-time field. Real solutions are formed as a real slice of the complex structure and determined by a real retarded-time parameter.

2 Complex structure of Kerr geometry and retarded-time construction.

Electrodynamic analogue of the Kerr solution was obtained by Appel in 1887 (!) [11]. A point-like charge e, placed on the complex Z-axis $(x_0, y_0, z_0) = (0, 0, ia)$ gives the Appel potential $\phi_a = Re\ e/\tilde{r}$, where $\tilde{r}$ is the Kerr complex radial coordinate $\tilde{r} = PZ^{-1} = r + ia\cos\theta$, and r, θ are the oblate spheroidal coordinates. It may be expressed in the usual Cartesian coordinates x, y, z, t as $\tilde{r} = [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2} = [x^2 + y^2 + (z - ia)^2]^{1/2}$, that corresponds to a shift of the source in complex direction $(x_o, y_o, z_o) \to (0, 0, ia)$, and can be considered as a mysterious "particle" propagating along a *complex* world-line $x_0^\mu(\tau)$ in CM^4.

The Kerr-Newman solution has just the same origin and can be described by means of a complex retarded-time construction as a field generated by such

a complex source [3, 7, 8, 9, 10].

Similar to the usual retarded-time scheme, a *complex* retarded time $\tau = t + i\sigma$ is determined by the family of complex light cones emanating from the points of the complex worldline $x_0^\mu(\tau)$.

The complex light cone with the vertex at some point x_0 of the complex worldline $x_0^\mu(\tau) \in CM^4$: $\quad (x_\mu - x_{0\mu})(x^\mu - x_0^\mu) = 0$, can be split into two families of null planes: "left" planes

$$x_L = x_0(\tau) + \alpha e^1 + \beta e^3 \tag{1}$$

spanned by null vectors $e^1(Y)$ and $e^3(Y, \tilde{Y})$, and"right"planes

$$x_R = x_0(\tau) + \alpha e^2 + \beta e^3, \tag{2}$$

spanned by null vectors e^2 and e^3.

The Kerr congruence $\mathcal{K}$ arises as a real slice of the family of the "left" null planes ($Y = const.$) of the complex light cones whose vertices lie on the complex worldline $x_0(\tau)$.

3 Kerr-Schild formalism and the Kerr theorem

In the Kerr-Schild backgrounds congruence $\mathcal{K}$ determines the ansatz

$$g_{\mu\nu} = \eta_{\mu\nu} + 2h e^3_\mu e^3_\nu, \tag{3}$$

where $\eta_{\mu\nu}$ is auxiliary Minkowski metric, and e^3 is principal null direction (PNC) of Kerr geometry. The PNC is null with respect to the auxiliary Minkowski spacetime as well as to the metric $g_{\mu\nu}$. [1]

The treatment via complex worldline can be related to the Kerr theorem by setting up a correspondence between parameters of the worldline and parameters of a generating function F of the Kerr theorem.

Traditional formulation of the Kerr theorem is as follows.

[1] We follow the notations of work [5] and use signature $(-+++)$. The Kerr-Schild null tetrad is completed as follows: $e^1 = d\zeta - Y dv, \quad e^2 = d\bar{\zeta} - \bar{Y} dv, \quad e^4 = dv + h e^3$. The dual tetrad e_a is determined by permutations: $e^1 \to e_2, e^2 \to e_1, e^3 \to e_4, e^4 \to e_3$. The tetrad derivatives are defined by $_{,a} \equiv \partial_a = e^\mu_a \partial_\mu$.

Any geodesic and shear-free null congruence in Minkowski space is defined by a function $Y(x)$ which is a solution of the equation

$$F = 0, \tag{4}$$

where $F(\lambda_1, \lambda_2, Y)$ is an arbitrary analytic function of the projective twistor coordinates $Y, \quad \lambda_1 = \zeta - Yv, \quad \lambda_2 = u + Y\bar{\zeta}$.

The Kerr congruence $\mathcal{K}$ is defined then by the vector field

$$e^3 = du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv \ , \tag{5}$$

where $2^{\frac{1}{2}}\zeta = x + iy, \quad 2^{\frac{1}{2}}\bar{\zeta} = x - iy, \quad 2^{\frac{1}{2}}u = z + t, \quad 2^{\frac{1}{2}}v = z - t$ are the null Cartesian coordinates.

In the Kerr-Schild approach the Kerr theorem acquires more broad contents [5, 7, 9]. It allows one to obtain the position of singular lines, caustics of the PNC, as a solution of the system of equations $F = 0; \quad dF/dY = 0$, and to define some important parameters of the solution:

$$\tilde{r} = - \quad dF/dY, \qquad P = \partial_{\lambda_1}F - \bar{Y}\partial_{\lambda_2}F. \tag{6}$$

The parameter $\tilde{r}$ characterizes a complex radial distance, and for the Kerr solution it is a typical complex combination $\tilde{r} = r + ia\cos\theta$. Parameter P is connected with the boost of the source.

Stationary congruences, having Kerr-like singularities contained in a bounded region, have been considered in papers [7, 12, 10]. It was shown that in this case function F must be at most quadratic in Y,

$$F \equiv a_0 + a_1 Y + a_2 Y^2 + (qY + c)\lambda_1 - (pY + \bar{q})\lambda_2, \tag{7}$$

where coefficients c and p are real constants and $a_0, a_1, a_2, q, \bar{q}$, are complex constants. Writing the function F in the form $F = AY^2 + BY + C$, one can find solutions of the equation $F = 0$ for the function $Y(x)$

$$Y_{1,2} = (-B \pm \Delta)/2A, \tag{8}$$

where $\Delta = (B^2 - 4AC)^{1/2}$. These two roots define two PNCs of the Kerr geometry. From (6) one has

$$\tilde{r} = -\partial F/\partial Y = -2AY - B = \mp\Delta, \tag{9}$$

and $P = pY\bar{Y} + \bar{q}\bar{Y} + qY + c$.

On the other hand, the stationary and boosted Kerr geometries are described by a straight complex worldline with a real 3-velocity $\vec{v}$ in CM^4:

$$x_0^\mu(\tau) = x_0^\mu(0) + \xi^\mu \tau; \qquad \xi^\mu = (1, \vec{v}) \ . \tag{10}$$

It was shown in [10] that parameters $p, c, q, \bar{q}$ are related to parameters of complex worldline $\partial_\tau x_0(\tau) = \xi^\mu$, or to the boost of the source. Meanwhile, the complex initial position of complex worldline $x_0^\mu(0)$ in (10) gives six parameters which are connected to coefficients $a_0, \ a_1 \ a_2$. It can be decomposed as $\vec{x}_0(0) = \vec{c} + i\vec{d}$, where $\vec{c}$ and $\vec{d}$ are real 3-vectors with respect to the space O(3)-rotation. The real part $\vec{c}$ defines the initial position of source, and the imaginary part $\vec{d}$ defines the value and direction of angular momentum.

4 Real slice and the real retarded time

In nonstationary case coefficients of function F turn out to be complex variable depending on the complex retarded-time parameter τ, and function $\partial_\tau x_0^\mu = \xi^\mu$ takes also complex values. The real slice of space-time is constructed from the "left" and "right"complex structures. The "left" structure is built of the left complex worldline x_0 and of the complex parameter Y generating the left null planes. The "right" complex structure is built of the right complex worldline $\bar{x}_0$, parameter $\bar{Y}$ and right null planes, spanned by vectors e^2 and e^3. These structures can be considered as functionally independent in CM^4, but they have to be complex conjugate on the real slice of space-time. For a real point of space-time x and for the corresponding real null direction e^3 we define a real function

$$\rho(x) = x^\mu e^3_\mu(x). \tag{11}$$

One can determine the values of ρ at the points of the left and right complex worldlines x_0^μ and $\bar{x}_0^\mu$ by L- and R-projections

$$\rho_L(x_0) = x_0^\mu e^3_\mu(x)|_L, \qquad \rho_R(\bar{x}_0) = \bar{x}_0^\mu e^3_\mu(x)|_R, \tag{12}$$

where the sign $|_L$ means that the points x and $x_0(\tau)$ are synchronized by the left null plane (1), and $x - x_0(\tau_L) = \alpha e^1 + \beta e^3$.

As a consequence of the conditions $e^{1\mu}e^3_\mu = e^{3\mu}e^3_\mu = 0$, we have

$$\rho_L(x_0) = x_0^\mu e^3_\mu(x)|_L = \rho(x). \tag{13}$$

So far as the parameter $\rho(x)$ is real, parameter $\rho_L(x_0)$ will be real too. Similarly, $\rho_R(\bar{x}_0) = \bar{x}_0^\mu e^3_\mu(x)|_R = \rho(x)$, and consequently, $\rho_L(x_0) = \rho(x) = \rho_R(\bar{x}_0)$. The parameters $\lambda_1, \quad \lambda_2$ can also be expressed in terms of the coordinates x^μ,

$$\lambda_1 = x^\mu e^1_\mu, \qquad \lambda_2 = x^\mu(e^3_\mu - \bar{Y}e^1_\mu). \tag{14}$$

It yields

$$\rho = \lambda_2 + \bar{Y}\lambda_1 \ . \tag{15}$$

The values of twistor parameters λ_1 and λ_2 can also be defined by L-projection for the points of the complex worldline, λ_1^0 and λ_2^0:

$$(\lambda_1 - \lambda_1^0)|_L = 0, \qquad (\lambda_2 - \lambda_2^0)|_L = 0. \tag{16}$$

The L-projection determines the values of the left retarded-time parameter $\tau_L = (t + i\sigma)|_L$. The real function ρ and the twistor variables λ_1 and λ_2 acquire an extra dependence on the retarded-time parameter τ_L. However, it should be noted that the real and imaginary parts of τ_L are not independent because of the constraint caused by L-projection. It means that *on the real slice* functions $\rho, \quad \lambda_1$ and λ_2 and functions $\rho_0, \quad \lambda_1^0, \quad \lambda_2^0$ can be considered as functions of the real retarded-time parameter $t_0 = \Re e \ \tau_L$.

On the other hand, in CM^4 function $t_0|_L$ is an analytic function of twistor parameters $Z^a = \{Y, \quad \lambda_1, \quad \lambda_2\}$ which satisfy the relation[2] $Z^a{}_{,2} = Z^a{}_{,4} = 0$. It has the consequence

$$t_{0,2} = t_{0,4} = 0 \ . \tag{17}$$

Similar to the stationary case considered in [7, 10], one can use for function F representation in the form

$$F \equiv (\lambda_1 - \lambda_1^0)K_2 - (\lambda_2 - \lambda_2^0)K_1, \tag{18}$$

where the functions K_1 and K_2 will be depending on the real retarded-time t_0 (or on the related real parameter ρ_0). It leads to the form (7) with the coefficients depending on the retarded-time.

[2] It can be obtained by direct differentiation. See also [7, 9]

Let us assume that the relation $F(Y,t_0)=0$ is hold by the retarded-time evolution $\partial F/\partial t_0|_L=0$. It yields

$$\frac{\partial F}{\partial t_0}=K_1\partial_{t_0}\lambda_2^0-K_2\partial_{t_0}\lambda_1^0+(\lambda_1-\lambda_1^0)\partial_{t_0}K_2-(\lambda_2-\lambda_2^0)\partial_{t_0}K_1=0. \quad (19)$$

As a consequence of (16), by L-projection last two terms cancel and one obtains

$$(\partial F/\partial t_0)|_L=(K_1\partial_{t_0}\lambda_2^0-K_2\partial_{t_0}\lambda_1^0)|_L=0, \quad (20)$$

that is provided by

$$K_1(t_0)=\partial_{t_0}\lambda_2^0, \qquad K_2(t_0)=\partial_{t_0}\lambda_1^0. \quad (21)$$

It seems that the extra dependence of function F on the real retarded-time parameter t_0 contradicts to the Kerr theorem; however, the analytic dependence of t_0 on $Y, \quad \lambda_1, \quad \lambda_2$ is reconstructed by L-projection. As a result, function F turns out to be analytic functions of twistor variables. Meanwhile, all the *real* retarded-time derivatives are non-analytic and have to involve the conjugate "right" complex structure.

As a consequence of the relation (6), one obtains

$$P=\bar{Y}K_1+K_2, \quad (22)$$

that yields for function P the real expression

$$P=\partial\rho_L/\partial t_0 \ =\Re e\ \partial_\tau(x_0^\mu(\tau)e_\mu^3|_L). \quad (23)$$

The coefficients A,B,C of the resulting decomposition $F=AY^2+BY+C$ will be functions of the retarded-time parameter t_0 as well as the solutions of (8),(9) defining $\tilde{r}$, $Y(x)$ and corresponding PNC.
[3].

5 Solution of the field equations.

For simplicity, we shall assume that there is no electromagnetic field. Similar to the Kinnersley case, we admit the existence of null radiation. Therefore,

[3]Note that the real function $\rho_L(t_0)$ plays the role of a potential for P, Similar to some nonstationary solutions presented in [2]

all the components of Ricci tensor R_{ab} have to be zero for the exclusion R_{33} that corresponds to an incoherent flow of the light-like particles in e^3 direction.

The process of the solution of the field equations is similar to the treatment given in [5]. In particular, we have $R_{24} = R_{22} = R_{44} = 0$.

If the electromagnetic field is zero, we have also $R_{12} = R_{34} = 0$, that leads to the equation $h,_{44} + 2(Z + \bar{Z})h,_4 + 2Z\bar{Z}h = 0$, which admits the solution

$$h = M(Z + \bar{Z})/2 \ , \tag{24}$$

where M is a real function, obeying the condition $M,_4 = 0$.

The equation $R_{23} = 0$, acquires the form

$$M,_2 - 3Z^{-1}\bar{Z}Y,_3 M = 0. \tag{25}$$

The last gravitational field equation $R_{33} = -P_{33}$ takes the form

$$\mathcal{D}M = Z^{-1}\bar{Z}^{-1}P_{33}/2, \tag{26}$$

where

$$\mathcal{D} = \partial_3 - Z^{-1}Y,_3 \partial_1 - \bar{Z}^{-1}\bar{Y},_3 \partial_2. \tag{27}$$

The term P_{33} is the contribution to energy-momentum tensor corresponding to the null radiation.

To integrate (25) we have to use the relation [4]

$$(\log P),_2 = -Z^{-1}\bar{Z}Y,_3 \ , \tag{28}$$

which allows us to represent (25) in the form

$$(\log MP^3),_2 = 0 \tag{29}$$

and to get the general solution

$$M = m/P^3, \tag{30}$$

where

$$m,_4 = m,_2 = 0. \tag{31}$$

[4]This relation was proved in [5] for stationary case. In nonstationary case function P has an extra dependence on t_0; however, because of (17) this relation is valid.

Since m has to be real, it can only be a function of t_0.

Action of the operator $\mathcal{D}$ on the variables $Y, \bar{Y}$ and ρ is as follows:

$$\mathcal{D}Y = \mathcal{D}\bar{Y} = 0, \quad \mathcal{D}\rho = 1. \tag{32}$$

From the last relation and (23) we have

$$\mathcal{D}\rho = (\partial\rho_L/\partial t_0)\mathcal{D}t_0 = P\mathcal{D}t_0 = 1, \tag{33}$$

which yields

$$\mathcal{D}t_0 = P^{-1}. \tag{34}$$

Since M is a function of $Y, \bar{Y}$ and t_0, equation (26) takes the form

$$\partial_{t_0} M = PZ^{-1}\bar{Z}^{-1}P_{33}/2. \tag{35}$$

This equation is the definition of the unique component P_{33} of the light-like radiation which propagates along twisting PNC. Substituting (30) one obtains

$$P_{33} = Z\bar{Z}[-6m(\partial_{t_0}P) + 2P(\partial_{t_0}m)]/P^3. \tag{36}$$

The first term describes a radiation caused by an acceleration of source while the second term corresponds to a radiation with a loss of mass that corresponds to the Vaidya "shining star" solution [2, 13, 14].

The resulting metric has the form $g_{\mu\nu} = \eta_{\mu\nu} + (m/P^3)(Z+\bar{Z})e^3_\mu e^3_\nu$. One can normalize e^3 by introducing $l = e^3/P$, and metric takes the simple form

$$g_{\mu\nu} = \eta_{\mu\nu} - m(\tilde{r}^{-1} + \bar{\tilde{r}}^{-1})l_\mu l_\nu, \tag{37}$$

where $\tilde{r} = PZ^{-1} = -dF/dY$.

The structure of this solution and the form of metric are similar to the Kinnersley solution [1] and correspond to its modification for the case of complex worldline.

Let us summarize the obtained solution. All the parameters are determined by a given complex worldline $x_0(\tau)$ and have the current values depending on the retarded-time parameter t_0 which is determined by L-projection

$$(\lambda_1 - \lambda_1^0)|_L = 0, \qquad (\lambda_2 - \lambda_2^0)|_L = 0. \tag{38}$$

The congruence

$$e^3 = du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv \tag{39}$$

is expressed in the null Cartesian coordinates $u, v, \zeta, \bar\zeta$ and is determined for a fixed retarded time t_0 by function

$$Y(x,t_0) = [-B + (B^2 - 4AC)^{1/2}]/2A \tag{40}$$

which is a solution of the equation $F = AY^2 + BY + C = 0$. The current parameters A, B, C are defined by this, quadratic in Y, decomposition of function F given by

$$F \equiv (\lambda_1 - \lambda_1^0)K_2 - (\lambda_2 - \lambda_2^0)K_1, \tag{41}$$

where

$$K_1(t_0) = \partial_{t_0}\lambda_2^0, \qquad K_2(t_0) = \partial_{t_0}\lambda_1^0. \tag{42}$$

The current function $\tilde r$ and P are given by

$$\tilde r = -2AY - B, \quad P = \bar Y K_1 + K_2. \tag{43}$$

If the worldline is real, $\Im m \ x_0 = 0$, the equation (30) takes the form $M(Y, \bar Y, t_0) = m/P^3$, where $P = \dot x_0^\mu e_\mu^3$ so that $\dot x_0^\mu l_\mu = 1$. We obtain $\tau_L = \tau_R$ that yields exactly the Kinnersley real retarded-time construction with metric $g_{\mu\nu} = \eta_{\mu\nu} + 2(m/r)(\sigma_\mu/r)(\sigma_\nu/r)$. The relation with our notations is $l^\mu = \sigma^\mu/r$, where $\sigma^\mu = x^\mu - x_0^\mu, \qquad r = PZ^{-1}$. The Kinnersley retarded-time parameter $u = \tau/\sqrt{(\dot x_0)^2}$, and the Kinnersley parameter $\lambda^\mu(u) = \dot x_0^\mu(\tau)/\sqrt{(\dot x_0)^2}$.

6 Conclusion.

The solutions considered represent a natural generalization of the Kinnersley class to the rotating case. The Kerr-Schild approach provides the exact retarded expressions for metric, coordinate system, PNC, and position of singularity in terms of the null Cartesian coordinates by arbitrary boost and acceleration of rotating source with arbitrary orientations of angular momentum.

The solutions obtained can find an application for modelling the radiation from the rotating astrophysical object by arbitrary relativistic boosts and accelerations and can also represent an interest for investigation of the role of gravitational field by particle scattering in ultrarelativistic regimes [10], as well as for modelling the Kerr spinning particle [7, 8, 16, 4], generating excitations of the string-like Kerr source [15]. On the other hand, the solutions

considered are direct generalizations of the rotating black hole solutions, and have the black hole horizons by the standard conditions $m^2 > e^2 + a^2$. Since the black hole solutions in the retarded-time scheme are based on outgoing principal null congruence, the black hole interior has to be considered in this case as the 'past'. The problem of radiation from black holes by acceleration demands a better understanding of the corresponding geometry and has to be considered elsewhere.

Some known generalizations of the Kerr solution, such as Kerr-Newman solution, Kerr-Sen solution to low energy string theory [15], solution to broken $N = 2$ supergravity [16] and the regular rotating BH-solutions [17], retain the form and (geodesic and shear-free) properties of the Kerr congruence. It means that the Kerr theorem can be used in these cases, and these solutions can also be generalized to the nonstationary cases.

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